T-1 Mechanics of Materials & Equation-of-State

Universality in the High-Pressure Hugoniot

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Data for the low-pressure shock Hugoniot is available for a large number of materials and the accompanying understanding of the physics is also in hand. Above 5 Mbar the number of data points is, relatively speaking, quite small with all useful data coming from nuclear driven experiments. An amalgamation of the high pressure information into a coherent, simple picture has been lacking.

Using the Hugoniot jump relations, which represent the conservation of mass, energy, and momentum through the shock process, one can derive

$$2S - 2 - \gamma = \frac{C}{SU_p} \left[1 - 3S + \frac{B_s}{P} - \frac{C}{U_p} \right].$$

The independent variable here is the shock particle velocity U_p . The thermodynamics enters through the isentropic bulk modulus B_s , the pressure P, and the Grüneisen constant

$$\gamma \equiv \frac{1}{\rho} \frac{\partial P}{\partial E} \bigg|_{\rho}.$$

All these are functions of U_p . The dependent variables are not the shock velocity U_s but instead are C and S. These are the intercept at $U_p = 0$ and slope of the tangent line to the $U_s - U_p$ Hugoniot, respectively. One is doing a Legendre transform. For strong shocks the right of the equation is small $(U_p$ is large) thus implying that

$$S = 1 + \gamma/2.$$

the slope of the $U_s - U_p$ curve is

For regions of the Hugoniot above 5 Mbar one can easily argue that ionization is the dominate physics of the equation of state and determines the behavior of γ . In particular it is expected that the energy sink of ionization will pull γ below the ideal gas value of 2/3. In addition a negative γ would be anomalous. Thus $\gamma = 0.4 \pm 0.1$ is a quite reasonable estimate

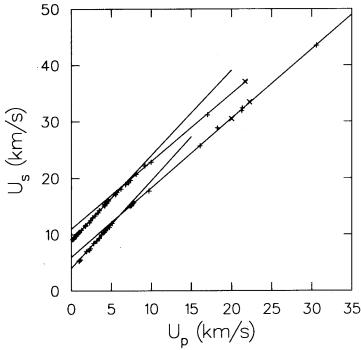


Figure 1: Fe and Cu Hugoniot data. We see clearly the linearity of the upper data and a well-defined break. The Cu data and curves have been shifted upwards.

throughout ionization; a large region of the Hugoniot. This implies that $S=1.2\pm0.05$ over a large range of U_p , from say 5 km/s up to a few hundred km/s. The slope is very constant, the U_s-U_p Hugoniot is linear.

Figures 1 and 2 show shock data for six elements Bi, Fe, Cu, Sn, Ar, and Xe. All exhibit the expected very linear $U_s - U_p$ curve for low pressure. The higher slope straight lines are fits to the low data. But we also see a break to a higher pressure region with very linear behavior. All these slopes fall between 1.14 and 1.22. This is all quite in line with electron ionization breaking the low-pressure Hugoniot over to another region of constant slope.

This new picture of the high-pressure Hugoniot gives us increased understanding, the capability to make estimates without data, and decreases the need for more high-pressure data.

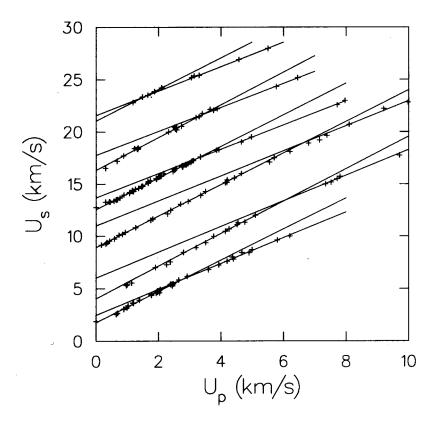


Figure 2: Bi, Fe, Cu, Sn, Ar, and Xe Hugoniot data from bottom to top. Shifts have been introduced to avoid overlaps.